A Note on the Textbook Phillips Curve

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ABSTRACT

The classical and more recent offshoot textbook Phillips Curve tradeoffs are re-investigated. An empirical analysis is done using annualized quarterly data from 1978 – 2009, which once again confirms there is no long run tradeoff between inflation and unemployment. Further, an empirical search for the short run textbook Phillips Curve is undertaken. Not surprisingly, there appears to be no statistically significant relationship between inflation and unemployment – even in the short run – over the past thirty or more years, and this is true whether the relationship is the classical one between inflation rate and unemployment rate; or the original Modigliani-Papademos NAIRU difference between current and lagged inflation rate and the gap between actual and the natural rate of unemployment; or the Friedman-Phelps-Lucas expectations-augmented one between the difference of actual and expected inflation rate and the gap between actual and the natural rate of unemployment. In search of other theoretically relevant variables, a continuously equilibrating labor market model is posited and solved for the standard Phillips Curve tradeoff. The resulting model is indeterminate for a unique unemployment rate without simplifying assumptions regarding the growth rates of both employment and labor force. Making those assumptions, the simple labor market model predicts that changes in the growth rates of GDP and employment may impact inflation independent of unemployment rates. The empirical analysis here, however, also fails to confirm statistically significant relationships for these two new variables.

Keywords: Phillips curve, Microfoundations, New Keynesian models, NAIRU, expectations augmented

INTRODUCTION

While the long run Phillips Curve was dismissed by the 'team' of Phelps (1967), Friedman (1968), and Lucas (1972) over forty years ago, the short run Phillips relationship between inflation rate and unemployment rate has seemingly endured. Its endurance is proved by the fact that it is commonly found in popular macroeconomic textbooks such as Olivier Blanchard's *Macroeconomics, 5e* (2006) or Richard Froyen's *Macroeconomics: Theories and Policies, 9e* (2009) and in well known money and banking textbooks such as Laurence Ball's *Money, Banking, and Financial Markets* (2009) or Frederic Mishkin's *Money, Banking, & Financial Markets, 2e* (2010).

The historical evolution of the Phillips Curve relationship from long run to short run and bringing in NAIRU has also been well documented. For instance, the Federal Reserve Bank of Richmond recently devoted an entire issue of its *Economic Quarterly* (2008) to the Phillips Curve and its implications for monetary policy, and in that *Quarterly* King (2008) gives the readers a well-researched historical tour. Lesser historical expositions can also be found in many of the textbooks listed above. But current approaches to Phillips curves in the economics discipline are usually of the Calvo-type or the Lucas Supply Curve type and are not like the form of the past. And yet, most macroeconomics textbooks still use the more classic 1960s and 1970s approaches, i.e.

(1) The Classic Phillips Curve: $(\pi_t) = f(U_t)$

(2) The simple adaptive expectations (Modigliani-Papademos, 1975) Phillips Curve:

$$(\pi_t) = f(\pi_{t-1}, U_t - U^*)$$

(3 The simple expectations augmented (Friedman, 1968-Phelps, 1967), sometimes now called a Barro-Gordon, Phillips Curve:

$$(\pi_t) = f(\pi_t^e, U_t - U^*)$$

The task in this paper, then, is simple: to determine whether any of these old, textbook models truly demonstrate a short run, textbook relationship between inflation and unemployment that is worthy of being in a textbook at all.

REGRESSION RESULTS FOR TEXTBOOK MODELS

An empirical analysis was performed for three standard forms of the textbook Phillips relation within different time periods between 1978 and 2009. In an attempt to compare and contrast models and time periods, simple linear (OLS) regressions were run as outlined below. Each decadal time period has three different models: the classic tradeoff between unemployment and inflation; the introduction of the non-accelerating inflation rate of unemployment (NAIRU, which introduces the output gap away from some natural rate of unemployment, and which itself is an offshoot of the Modigliani-Papademos (1975) NIRU as Espinosa-Vega and Russell (1997) rightly point out) with simple adaptive expectations; and finally, the Friedman-Phelps-Lucas rational expectations approach to NAIRU.

EMPIRICAL RESULTS OF TEXTBOOK MODELS THROUGH THE 1980s

The following simple OLS regressions were run with quarterly 1980s data for a total of 40 observations. The data were culled from the Federal Reserve Bank of St. Louis's Federal Reserve Economic Data (FRED) and Michigan Survey except for natural rates of unemployment, which were gleaned from Robert Gordon's (2006) *Macroeconomics 10e*. All data are in annualized percentage form and standard t-stats are in parentheses below the coefficients.

The Classic Phillips Curve: $(\pi_t) = f(U_t)$.

(4) $$\pi_t = 6.81387 - \underset{(-0.5303)}{0.22478}\, U_t; \qquad R^2 = 0.0073$$

The simple adaptive expectations Phillips Curve: $(\pi_t) = f(\pi_{t-1}, U_t - U^*)$.

(5) $$\pi_t = 2.3589 + \underset{(4.384)}{0.5653}\, \pi_{t-1} - \underset{(-0.6509)}{0.2256}\, (U_t - U^*) \qquad R^2 = 0.345$$

The simple expectations augmented Phillips Curve: $(\pi_t) = f(\pi_t^e, U_t - U^*)$.

(6) $$\pi_t = -0.225 + \underset{(9.405)}{0.3472}\, \pi_t^e - \underset{(-1.338)}{0.07763}(U_t - U^*) \qquad R^2 = 0.7066$$

Even a cursory look at these 1980s regressions will reveal that neither unemployment rates nor gaps in unemployment rates have any impact on inflation. No statistical significance is seen for any of the three equations. Of the variables used for the empirical analysis, only expected inflation shows strong statistical significance as an explanatory variable for inflation.

EMPIRICAL RESULTS OF TEXTBOOK MODELS THROUGH THE 1990s

The following regressions were run with quarterly 1990s data for a total of 40 observations. The data were also culled from the Federal Reserve Bank of St. Louis's FRED and Michigan Survey except for natural rates of unemployment, which were gleaned from Robert Gordon's *Macroeconomics 10e*. All data are in annualized percentage form and standard t-stats are presented in parentheses below the coefficients.

The Classic Phillips Curve: $(\pi_t) = f(U_t)$.

(7) $$\pi_t = 1.556078 + \underset{(.9316)}{0.24022}\, U_t; \qquad R^2 = 0.0223$$

The simple adaptive expectations Phillips Curve: $(\pi_t) = f(\pi_{t-1}, U_t - U^*)$.

(8) $\pi_t = 2.7325 + 0.0745\,\pi_{t-1} - 0.0873\,(U_t - U^*);$ $R^2 = 0.0067$
(.4528) (−0.2556)

The simple expectations augmented Phillips Curve: $(\pi_t) = f(\pi_t^e, U_t - U^*)$.

(9) $\pi_t = -1.42186 + 0.7159\,\pi_t^e - 0.0282(U_t - U^*)$ $R^2 = 0.6095$
(7.5916) (−0.241)

Once again, neither unemployment rates nor gaps in unemployment rates appear to have any correlation with inflation during the 1990s. And once again, just as through the 1980s, only expected inflation shows strong statistical significance as an explanatory variable for inflation.

EMPIRICAL RESULTS OF TEXTBOOK MODELS THROUGH THE 2000s

The following regressions were run with quarterly 2000s data. Some regressions have been run with more data than others due to a lack of data for natural rates of unemployment past 2004. Each regression reports the time periods used. The data were once again culled from the Federal Reserve Bank of St. Louis's FRED and Michigan Survey except for natural rates of unemployment, which were gleaned from Robert Gordon's *Macroeconomics 10e*. All data are in annualized percentage form and standard t-stats are presented in parentheses below the coefficients.

The Classic Phillips Curve: $(\pi_t) = f(U_t)$ (from 2000.1– 2009.2)

(10) $\pi_t = 8.3373 - 1.10564\,U_t;$ $R^2 = 0.0806$
(−1.1776)

The simple adaptive expectations Phillips Curve: $(\pi_t) = f(\pi_{t-1}, U_t - U^*)$ (from 2000.1 – 2004.4)

(11) $\pi_t = 3.7796 - 0.42065\,\pi_{t-1} - 0.8848\,(U_t - U^*);$ $R^2 = 0.217$
(−1.887) (−1.5657)

The simple expectations augmented Phillips Curve: $(\pi_t) = f(\pi_t^e, U_t - U^*)$ (from 2000.1 – 2004.4)

(12) $\pi_t = -0.9927 + 0.2008\,\pi_t^e + 0.0598(U_t - U^*)$ $R^2 = 0.386$
(3.0356) (0.434)

The 2000s mimic the trend seen earlier with neither unemployment rates nor gaps in unemployment rates being shown to have any apparent connection with inflation, with the exception that in the simple adaptive expectations Phillips Curve the t-stat for the unemployment gap is hovering closer to statistical significance (p-value was .076). And once again, just as through the 1980s and 1990s, only expected inflation shows a strong statistical significance as an explanatory variable for inflation.

A MICROECONOMICS DERIVATION OF THE PHILLIPS RELATION

The empirical investigation of these several different forms of the textbook Phillips curves suggests that much of the excitement about the textbook relationship between inflation and unemployment is unwarranted, even in the short run. Perhaps other variables are missing or there really is no economic relationship? To find out, a Phillips relation is derived from an equilibrating labor market which confirms that, as would be expected, no long run Phillips curve exists in this framework. More importantly, two variables are found to theoretically impact the short run Phillips relation but have been omitted from most theoretical and empirical studies. It is found that changes in the speed of adjustment for these variables to equilibrate the labor market may significantly impact the relationship between unemployment and inflation.

A LABOR MARKET-BASED MODEL

To begin a very simple, linear, constant equilibrium model of the labor market is posited. This model determines a long run Phillips relation and consequently does not reflect a connection between inflation and unemployment. It does predict, however, that a relationship between inflation and the *rate of change* of unemployment should exist. The labor market model is developed below.

(13) $W^d = P\left(\frac{dY}{dN}\right)$

(14) $W^s = W_0 + \alpha_0 N - \alpha_1 LF + \alpha_2 P^e$, and

(15) $W^d = W^s = W^*$

where W^d is nominal wage demand, W^s is the nominal wage supply, P is the price level, Y is real income, N is the level of employment, LF is the labor force and P^e is the expected price level. Placing the variables in log form and taking the time derivative yields rates of change for the variables, which are shown in lower case except the inflation rate has been changed to π in keeping with most of the literature:

(16) $w = \pi + (dy) - (dn)$, from equation (13), and

(17) $w = w_0 + \alpha_0 + n - \alpha_1 - lf + \alpha_2 + \pi^e$, from equation (14).

Setting (16) and (17) equal and solving for π gives

(18) $\pi = w_0 + \alpha_0 - \alpha_1 + \alpha_2 + (dn) - (dy) + \pi^e + [n - lf]$

Looking only at the last two terms, those in brackets in (18), the expression may be rewritten as

(19) $\quad -[lf - n] = -u$, where u is the rate of change of unemployment, and which in discrete time form is

(20) $$-\left[\left(\frac{LF_t - LF_{t-1}}{LF_{t-1}}\right) - \left(\frac{N_t - N_{t-1}}{N_{t-1}}\right)\right] = -\left(\frac{U_t - U_{t-1}}{U_{t-1}}\right)$$

Finally, from (18) and (19) is obtained

(21) $$\pi = \varphi + (dn) - (dy) + \pi^e - u$$

where $\varphi = w_0 + \alpha_0 - \alpha_1 + \alpha_2$. Ideally, (21) would contain the expression $\left(\frac{LF_t - N_t}{LF_t}\right)$ rather than – u, that is, it would contain the discrete form of the unemployment rate; however, the discrete form is not mathematically derivable from the rate of change form, even with attempts at simplifying assumptions. For instance, holding labor force participation and employment rates of change constant forces the unemployment rate to be constant as well, which negates that possibility. And the problem with the simple model cannot be resolved by adding two more equations, one for each of those two variables, labor force participation and employment. These types of models already exist in the literature, the classic being Pissarides (1985); however, search and matching models are not presented as part of the textbook Phillips relation but are instead used as the backbone of Beveridge curves (see, among many instances, Valletta (2005) or Bouvet (2009)) and are therefore left out of the standard Phillips Curves that are considered here.

The expression (21) is in simple linear form and is somewhat like the textbook Phillips models except the unemployment 'rate' (which is in actuality a proportion) is replaced by the rate of change of unemployment. The slope term is unambiguously negative in this form, but again, the form here does not match the standard Phillips relation. All of the intercept terms are the shift components for labor supply and demand, so if φ changes the corresponding inflation rate will rise or fall based on the sign of the coefficient and time of adjustment.

Long run insight

In equation (21), the usual Friedman-Phelps-Lucas long run result seems to be there, that an increase in expected inflation will shift the curve upward in a 1-1 ratio, i.e., expected inflation is equal to actual in the long run. A reasonable test, then, is to check for the inflation expectations coefficient to be unitary. The empirical results below, however, do not show anything near a 1-1 ratio; on the other hand, it is true that the expected result doesn't come about in the standard textbook models either. Also, the conventional result that autonomous upward shifts of the labor supply curve (shown by w_0) will shift the long run Phillips Curve upward is included as well. Finally, the ambiguous slope is a reflection of the long run Phillips Curve being vertical, which is empirically supported here.

Short run insight

Some of (21) is new. For instance, α_0, the wage response to increases in employment, shows a positive impact on inflation as the response rate increases; α_1, the wage response to increases in the labor force, shows a negative impact on inflation as the response rate increases;

similarly, the wage response to increases in expected inflation, α_2, also pushes inflation higher as the coefficient rises. But more importantly, as the rate of employment speeds up, $(dn) > 0$, this has a positive impact on inflation, and as real GDP grows at a faster rate, $(dy) > 0$, this has a negative inflationary impact.

One of these last two results has been long known from the famous equation of exchange but seems to have been forgotten or pushed aside as the economic fraternity has moved to the standard NAIRU view. To reiterate, theory suggests that, *ceteris paribus*, faster growing GDP *lowers* the rate of inflation, it does not increase it. It is confirming that this same conjecture, well known from the equation of exchange, is derived from a completely different theoretical underpinning.

A new theoretical conjecture developed here is that, *ceteris paribus*, faster growing employment *increases* inflation, it does not lower it. To the extent that these two variables, i.e. GDP growth rates and employment growth rates, are positively correlated and cointegrated but have opposite impacts, the final reflection on inflation remains to be seen. However, inasmuch as both variables are shown to have separate impacts on inflation, the variables have been added to the modern expectations-augmented Phillips Curve model used earlier. The empirical results are below.

EMPIRICAL RESULTS OF THE MODEL DEVELOPED HERE

The expectations augmented Phillips Curve with the two variables developed here added, i.e. $(\pi_t) = f(\pi_t^e, dy, dn, U_t - U^*)$:

1980s:

$$(22) \quad \pi_t = \underset{(9.405)}{-1.406 + 1.475\,\pi_t^e} - \underset{(-0.85)}{27.0879\,dy_t} + \underset{(1.343)}{1.307\,dn_t} - \underset{(-0.926)}{0.2168\,(U_t - U^*)} \qquad R^2 = 0.732$$

1990s:

$$(23) \quad \pi_t = \underset{(6.7608)}{-5.7018 + 2.860\,\pi_t^e} - \underset{(-1.9088)}{58.1749\,dy_t} - \underset{(-0.1011)}{0.096\,dn_t} + \underset{(0.0907)}{0.0191\,(U_t - U^*)} \qquad R^2 = 0.650$$

2000.1 – 2004.4:

$$(24) \quad \pi_t = \underset{(3.017)}{-7.3669 + 3.6094\,\pi_t^e} - \underset{(-0.8607)}{49.2233\,dy_t} - \underset{(-1.3403)}{3.296\,dn_t} + \underset{(1.0063)}{0.6224\,(U_t - U^*)} \qquad R^2 = 0.462$$

Outside of the decade of the 90s, when dy_t is statistically significant and has the proper sign, no other time periods or variables are statistically relevant to inflation besides the usual inflation expectations variable. Change in employment is never statistically significantly different from zero, while unemployment gaps are also consistently irrelevant to explaining inflation. It can be presumed that this study fails to bring clear evidence of prior variable omission bias in the usual textbook approaches.

SUMMARY AND CONCLUSIONS

It is apparent that for over thirty years there has been no statistically significant relationship between any of the usually mentioned variables associated with a Phillips Curve. Indeed, the model derived here also fails to bring to light any new empirically relevant variables. Only expectations of inflation consistently showed statistical significance irrespective of the models that were tested. Still, the newest textbooks in the field such as Fontana and Setterfield's *Macroeconomic Theory and Macroeconomic Pedagogy* (2010) or Bain, Keith and Peter Howells, *Monetary Economics, Policy and its Theoretical Basis, 2e* (2009) use a three-equation supposed, "new consensus," New Keynesian model that incorporates the Phillips Curve as one of the three equations. In fact, Charles Jones' *Macroeconomics, 2e* (2011) even has a chapter (12) entitled "Monetary Policy and the Phillips Curve." Why? Using models that are known to have no empirical or theoretical basis has no place in determining policy and no place in a macroeconomics course.

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APPENDIX: A VISUAL APPROACH – SCATTER PLOTS

Often a picture is worth a thousand words (or regressions), so three scatter plots, showing quarterly data from the late 1970s to the 2000s are included in the appendix below to show the types of functional relationships that were empirically investigated here.

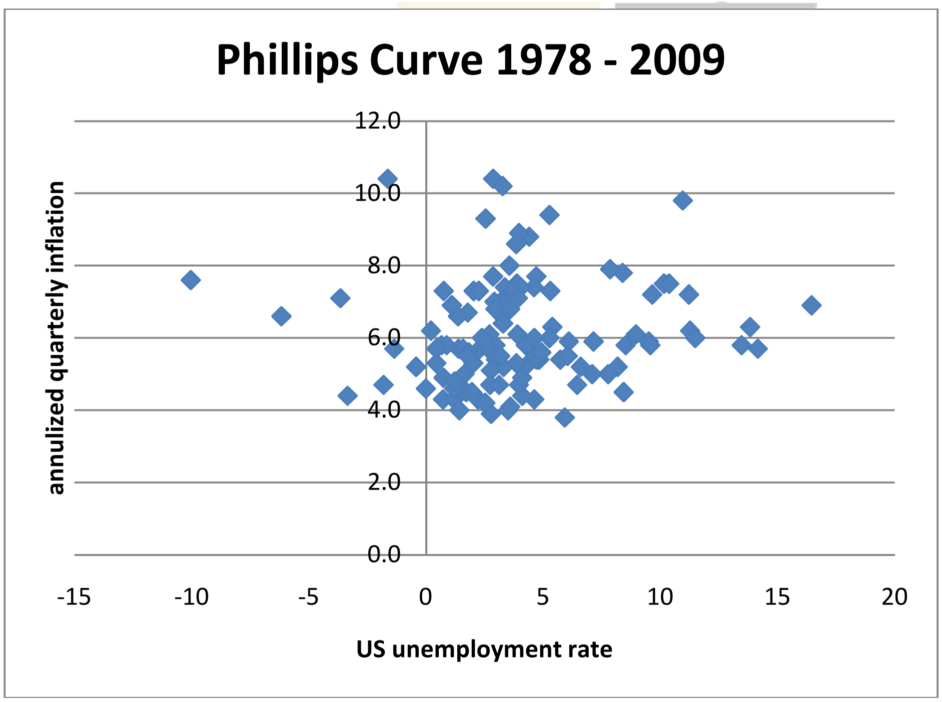

Figure 1 A graphical depiction of $(\pi_t) = f(U_t)$ for the period, 1978 - 2009

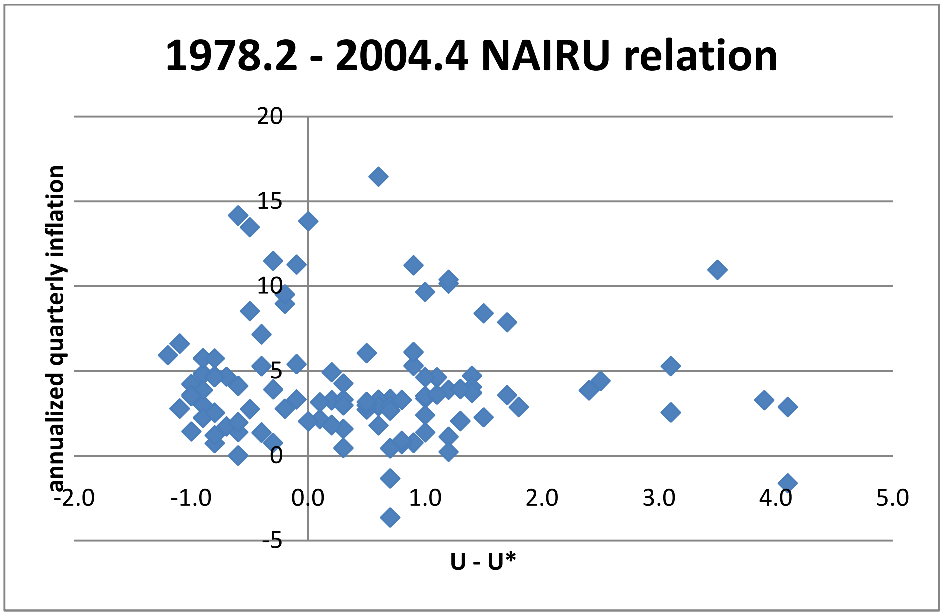

Figure 2 A graphical depiction of $(\pi_t) = f(U_t - U^*)$ **for the period, 1978 - 2009**

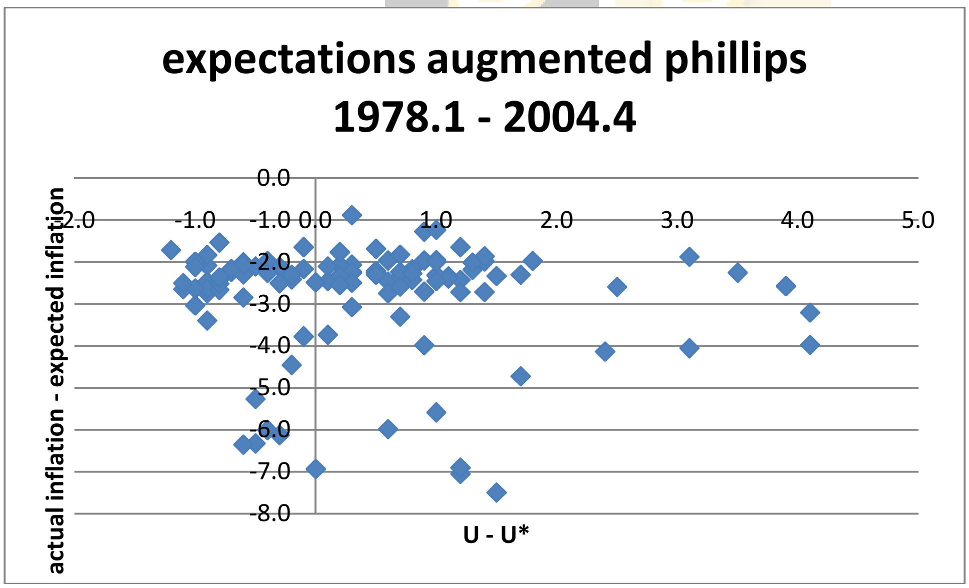

Figure 3 A graphical depiction of $(\pi_t - \pi_t^e) = f(U_t - U^*)$ **for the period, 1978 - 2004**