On the functional forms and stability of money demand: the U.S., Japan and Australia

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ABSTRACT

This study provides strong evidence for the stability of money demand of a semi-log form for the U.S., a log-log form for Japan and a semi-log form for Australia. This implies that there could be no finite satiation on the money demand curve as the nominal interest rate approaches zero and continue to stay near zero for a long period of time. The results support Friedman's (1969) zero nominal interest rate rule. The welfare cost of inflation using the functions derived by Lucas (2000) from Bailey's (1956) definition of the welfare cost is also estimated.
JEL: E41

Keywords: money demand, cointegration, welfare cost of inflation

INTRODUCTION

Lucas (2000) argues that the log-log specification of Meltzer's (1963) money demand provides a better fit in the U.S. data than the semi-log specification of Cagan's (1956); hence, calling for the Federal Reserve to abandon the low-but-positive inflation policy and adopt the optimal monetary policy of Friedman's (1969) zero nominal interest rate rule. A recent paper by Ireland (2009), however, pointed out to an important caveat to Lucas' (2000) conclusion. Ireland (2009) shows the opposite that the interest semi-log form of Cagan (1956) fits better with the post-1980 quarterly data.

The two specifications of the long-run relationship between money demand and interest rate are written as:

Meltzer (1963):

$$\ln(m) = \ln(A) - \eta \ln(r) \tag{1}$$

Cagan (1956):

$$\ln(m) = \ln(B) - \xi r \tag{2}$$

where m denotes the money-income ratio and the nominal interest rate (r) represents the opportunity cost of money. $\eta > 0$ and $\xi > 0$ measure the interest elasticity and interest semi-elasticity of money demand, respectively.

As emphasized by Lucas (2000), the different functional forms of money demand have very different behavior at low interest rates; hence, very different implications for the welfare cost[i] of inflation when the central bank moves to the Friedman's (1969) rule. The tail of the curve of the log-log specification of money demand implies that as interest rate approaches zero, the real money balances can become arbitrarily larger, while the semi-log specification has a finite satiation point (the intercept). Therefore, getting the right functional form of money demand is the first important step in evaluating the implementation of the monetary policy.

However, evaluating the stability of the specification is another important step to follow. It is questionable to whether each specification is stable over time or it may change as the nominal interest rate approaches zero and remains at near zero for a longer period of time. There are significant differences in the dataset used by Lucas (2000) and Ireland (2009) to explain their different results. The better-fit log-log specification of Lucas may result from the fact that the data contain a long period of low nominal interest rates from the 1930s to mid 1950s, whereas Ireland's estimation sample covers a period of high nominal interest rates, except a brief period of low interest rates from 2002 to 2004. The specification dynamic is the focus of this study.

On the other hand, another essential part is looking for the right measure of monetary aggregate. Because of the retail sweep programs adopted by commercial banks since 1994, Ireland (2009) uses M1RS as a measure of money stock. M1RS is computed for the years since 1994 by adding M1 with the retail sweep funds estimated by Cynamon, Dutkowsky and Jones (2006). At the same time, Carlson and Keen (1996), Carlson et al. (2000) and Teles and Zhou (2005) argue that the retail sweep programs together with banking deregulation in the 1980s and the improvement in electronic payments in the 1990s has replace M1 with the money zero maturity (MZM)[ii] in the role of money as liquid assets. Carlson and Keen (1996) and Carlson et al. (2000) provide evidence of the stability of the demand for MZM in the 1990s while Teles and Zhou (2005) estimate the money demand of the log-log specification and report the stability of the interest elasticity at 0.24. For the U.S. data, assessing the validity of the two measures is another focus of the paper.

The main objective of the paper is to evaluate the functional forms and stability of money demand at a low versus a high interest rate environment. The estimations are performed for three countries, the U.S., Japan and Australia for the post-1980 data. These countries are unique in the interest rate movements. The nominal interest rate in Japan has remained below 0.1 percent since mid 1995 while the cash rate in Australia has almost reached 3 percent over 3 decades and the U.S. Fed funds rate has just fallen to near zero since late 2008. The study is warranted because the phenomenon related to functional forms and stability of money demand at a low versus a high interest rate environment is limited in the current literature. Looking at the different behaviors of money demand in these three countries would provide an important implication of the satiation point of money demand as well as the comparative implication for the dynamic of the specification. Then, the welfare cost of inflation under each country's specification is evaluated for policy implications.

The results provide strong evidence for the stability of the demand for M1RS in a semi-log form for the U.S., the demand for M1 in a log-log form for Japan and the demand for M1 in a semi-log form for Australia. Whereas the semi-log form is stable for the country with high nominal interest rates, the log-log specification for the country where nominal interest rates remains near zero implies that there is no satiation point on the money demand curve. At the same time, the remaining question for the U.S. is whether the specification will change as the interest rates remain very low for a longer period of time.

The welfare cost of inflation using the functions derived by Lucas (2000) from Bailey's (1956) definition of the welfare is estimated. The results indicate that at the 6 percent nominal interest rate or 3 percent inflation, the welfare cost as opposed to the Friedman (1969) rule of zero interest rate is 0.05 percent for the U.S., 0.20 percent for Japan and 0.70 percent for Australia. Moreover, the gain from moving from a 10 percent inflation rate to zero inflation or price stability is equivalent to an increase in real income of 0.20 percent for the U.S., 0.30 percent for Japan and 2.55 percent for Australia. The lower gain for the U.S. is due to satiation on the money demand.

METHOD AND DATA

This study employs the ideas of cointegration to examine the stable relationship between money and interest rate. Phillips-Ouliaris (1990) test for cointegration is applied while the nonstationarity of the series are tested using Phillips-Perron (1988) unit root test. To test for the stability of the long-run money demand functions over time, a similar strategy used by Hoffman and Rasche (1991), Hoffman, Rasche and Tieslau (1995) and Stock and Watson (1993) is adopted. Specifically, this paper follows the functional form and the right measure of money demand by exploring whether the cointegration of each specification under each measure prevails in the recursive samples where the starting point of the sample period is held fixed and ending the estimation by updating the sample by 4 quarters recursively. This paper also employs Stock and Watson's (1993) dynamic ordinary least squares (DOLS) estimation to confirm that the results of the static estimates are robust.

The data used in this paper are quarterly series and the variables for monetary aggregates and gross domestic products (GDP) of all countries are seasonally adjusted. For the U.S., all variables are extracted from the Economic Data - FRED of the Federal Reserve Bank of St.

Louis. The money stock is measured by Money Zero Maturity (Series MZMSL) and M1RS which is M1 plus retail sweep funds estimated by Cynamon, Dutkowsky and Jones (2006) and nominal gross domestic product (GDP) measures income. The difference between the 3-month Treasury bill rate (Series TB3MS) and MZM own rate measures the opportunity cost of holding MZM while the 3-month Treasury bill rate measures the opportunity cost of holding M1RS. The GDP deflator is used to convert the variables from nominal to real. The time period spans from 1980Q1 to 2008Q3 for the demand for MZM and from 1980Q1 to 2010Q2 for the demand for M1RS. The variable measuring the opportunity cost of MZM becomes negative after the third quarter of 2008; hence, it cannot be use in the log form. The variables in the MZM equation are very closely comparable to Teles and Zhou (2005) while those in the M1RS equation resemble Ireland (2009).

The Japanese demand for money balance is measured by M1 (Series MA'MAMS5AAM1X12) and the opportunity cost of money is proxied by the collateralized overnight call rates (Series ST'STRACLCOON). These two series are taken from the website of the Bank of Japan. The measure of income is GDP estimate (Series 93SNA) and GDP deflator is used to convert from nominal to real. The data are taken from the website of the Cabinet of Office, Japan. The series range from 1980Q1 to 2010Q2.

The money stock for Australia is also measured by M1 (Series DMAM1S) which is extracted from the website of the Reserve Bank of Australia. The interbank cash rate (Series FIRMMCRI) measures the nominal interest rate. The cash rate is chosen over the 1-month or 3-month Treasury note rate because there were no Treasury notes issued between May 2002 and March 2009; hence, a series break. However, for the rest of the years, the three rates are very marginally different. Again, the income is measured by GDP (Series A2304418T) which is taken from the Australian Bureau of Statistics. The GDP deflator downloaded from the International Financial Statistics (IFS) CD-ROM 2011 is used to convert the nominal variables into the real ones. The series run from 1980Q1 to 2010Q2.

Figure 1 displays the scatter plots of the relationship between money-income ratio and its opportunity cost of holding money for the three countries. In the U.S. data, M1RS-GDP ratio and MZM-GDP ratio are plotted side by side. The two measures seem to behave quite differently at very low interest rates. At near zero interest rates, the demand for M1RS seems to reach a finite satiation while the demand for MZM arbitrarily increases. Apparently, the demand for M1RS seems to pick up the semi-log form while the demand for MZM seems to behave like the log-log specification.

The Japanese data seem to fit with the log-log form of money demand function. At the same time, another notable feature when comparing the Japanese demand for M1 and the U.S. demand for M1RS is observed. Looking at the Japanese data for the period before the call rate hit its near zero level (the pre-1995 period), the relationship between the M1-GDP ratio and the call rate resembles quite closely the current relationship between M1RS-GDP ratio and the Treasury bill rate. And the demand for M1 arbitrarily rises as the call rate remains longer at the near zero level. That the demand for M1RS will behave in a similar fashion is a further investigation. And, the Australian data seem not to give a clear picture of which functional form it may take.

ESTIMATION RESULTS

Most of the empirical literature on money demand has revolved around the ideas of nonstationarity and cointegration. That is to check if there exists a linear combination linking the nonstationary variables. Following Anderson and Rasche (2001), equation (1) and (2) are treated as a linear relationship between $\ln(m)$ and $\ln(r)$; and $\ln(m)$ and r, respectively. In this study, the analysis applies Phillips and Perron (1988) unit root test on $\ln(m)$, $\ln(r)$ and r separately, treating $\ln(r)$ and r as two different variables.

Table 1 and 2 report the results of the unit root test to each of the three variables, $\ln(m)$, $\ln(r)$ and r for the U. S., Japan and Australia. Specifically, an OLS estimation is performed for each variable on a constant (μ) and its own lagged value (ρ is the slope coefficient). The null hypothesis that the series has a unit root $(H_0: \rho = 1)$ is tested. Normally, the error term is serially correlated; hence, the Phillips-Perron test statistic Z_t is computed by using Newey and West (1987) estimator of its long-run variance. This study allows the positive autocorrelations in the error term (the lag truncation q) to go from 0 to 8 quarters. The critical values for Z_t is reported in Hamilton's (1994, p. 763) Table B.6.

For the U.S., both measures of money balances and both measures of the opportunity cost of holding money balances are tested (Table 1). The null hypothesis of a unit root is rejected for almost all variables in the M1RS demand equation. Thus, the two functional forms of money demand can be tested for cointegration. For all variables in the MZM demand equation, the results reject the null hypothesis of unit root for the log of MZM and log of interest rate, but fail to reject the null for the interest rate at the level. Thus, only the log-log specification of demand for MZM is tested for cointegration.

In Table 2, none of the test statistics can reject the null hypothesis of a unit root for the three variables of Japan and Australia. This allows for tests of cointegration between pairs of these apparently nonstationary variables.

MONEY DEMAND EQUATION: THE UNITED STATES

The Phillips and Ouliaris (1990) test for cointegration is applied. The approach starts by estimating an OLS regression of equation (1) linking the nonstationary variable $\ln(m)$ and $\ln(r)$ or (2) linking the nonstatonary variable $\ln(m)$ and r, then applying the Phillips-Perron's unit root test on the regression error. If the error term is stationary, then there exists a cointegrating relationship between two nonstatoinary variables.

Table 3 presents the results of the cointegration tests for the U.S. using M1RS as a measure of money stock and the 3-month Treasury bill rate as a measure of opportunity cost of holding money. The sample estimation starts with the period from 1980:1-2000:4 and is extended by 4 quarters recursively until 2010:2. The table reports the intercept $(\hat{\alpha}_1)$ and slope coefficients $(\hat{\alpha}_2)$ from a linear regression of equation (1) in the left panel and (2) in the right panel, the slope coefficient $(\hat{\rho})$ from a regression of the error term from equation (1) or (2) on its own lagged value, and the Phillips-Ouliaris statistic Z_t for values of the Newey-West lag truncation parameter q = 4 or 8 (the positive autocorrelation is allowed for 1 or 2 years.) Critical values for Z_t are provided under "Case 2" in Hamilton's (1994, 766) Table B.9.

The results confirm the stability of Ireland's (2009) preferred semi-log specification to Lucas' (2000) log-log specification in the post-1980 data. None of the tests summarized in the left panel of Table 3 rejects the null hypothesis of no cointegration between $\ln(m)$ and $\ln(r)$ throughout the recursive samples. On the other hand, the tests summarized in the right panel of Table 3 rejects the null hypothesis of no cointegration between $\ln(m)$ and r starting from the recursive sample at 1980-2002 with lag truncation parameter q = 8 at the 10 percent level and with both lag truncation q = 4 and 8 for the samples extended to 2005 through to 2010. The results provide statistical evidence of the stability of the semi-log money demand specification for the post-1980 data. The estimated interest semi-elasticity is very stable ranging from 1.55 to 1.67 across the sample lengths which is slightly smaller than Ireland's (2009) estimate of 1.79.

Table 4 presents the results of the cointegration tests for the U.S. using MZM as a measure of money stock and the spread between the 3-month Treasury bill rate and MZM own rate as a measure of opportunity cost of holding money. The table only displays the log-log specification because *r* is stationary; hence, the cointegration test is not applicable for the semi-log form. Compared to the results obtained in the left panel of Table 3, the point estimates of interest elasticity is about twice as large. However, the test statistics fail to reject the null hypothesis of no cointegration in all the sample lengths. The results provide statistical evidence of the instability of the log-log specification of money demand for MZM. Taken together with the results obtained in Table 3, the evidence seems to reject the claim by Carlson and Keen (1996), Carlson et al. (2000) and Teles and Zhou (2005) that MZM is the right measure of monetary aggregate in the money demand function after the banking deregulation and financial innovation in the 1980s and 1990s.

Note that the results obtained in Table 3 and 4 are estimated from equation (1) and (2) where unitary income elasticity is imposed so that the variable on the left hand side ln(*m*) is the money-income ratio. As a robustness test to confirm that the results of no cointegration between ln(*m*) and ln(*r*) are not driven by the imposition of unitary income elasticity, restriction is relaxed by estimating a linear relationship linking real money balances ln(*M/P*) to real income ln(*Y/P*) and nominal interest rate ln(*r*). Table 5 (see Web Appendix)[iii] reports the results of the money demand relationship for M1RS on the left panel and MZM on the right panel.

The table reports the OLS estimates of the intercept $\hat{\beta}_1$ together with the slope coefficients $\hat{\beta}_2$ and $\hat{\beta}_3$ that measure the income and interest elasticities of money demand, respectively. As before, the slope coefficient ($\hat{\rho}$) is obtained from the regression of the error term on its own lagged value, and the Phillips-Ouliaris statistic Z_t is calculated by using Newey and West (1987) estimator of its long-run variance with lag truncation parameter q = 4 or 8. Due to the nonzero drift in the explanatory variable ln(*Y/P*), critical values for Z_t provided under "Case 3" in Hamilton's (1994, 766) Table B.9 is used.

In both panels of Table 5, the point estimates of income elasticity $\hat{\beta}_2$ exceed unity and rise when the samples are recursively updated while the point estimates of interest elasticity seem to behave the opposite. In addition, comparing the estimates across the panel, both income and interest elasticities of money demand for MZM are larger. Nevertheless, none of the test statistics Z_t in both panels rejects the null hypothesis of no cointegration throughout the sample lengths. Therefore, more evidence points to M1RS as the right measure of monetary aggregate and its functional form stability in predicting the consequences of monetary policy. Our findings here confirm the argument by Ireland (2009) that the U.S. money demand function has changed for the post-1980 period.

To guarantee that the coefficient estimates and their stability across the sample lengths are not driven by the estimation technique, this paper employs a Stock and Watson (1993) dynamic ordinary least squares (DOLS) as another robustness test. Stock and Watson (1993) demonstrate that the dynamic OLS estimates are asymptotically efficient and equivalent to the estimates obtained by the maximum likelihood estimation (MLE) of Johansen (1988). Unlike the static regressions, the approach cannot be used to test the hypothesis of cointegration or no cointegration, but requires prior knowledge of the cointegrating relationship between the nonstationary variables. Hence, only the semi-log money demand specification linking ln(*m*) and

r is estimated.[iv] Specifically, this study adds two leads and lags of Δr to equation (2) and estimates the regression by using the OLS method. The procedure is to control for possible correlation between the interest rate r and the error term from the cointegrating relationship linking ln(m) and r. The paper uses the lag truncation q = 4 and 8 to account for the autocorrelation in the error term from the DOLS estimation in calculating the standard error of the estimate. The results are reported in Table 6 (see Web Appendix).

The table reports the intercept $(\hat{\alpha}_1)$ and the slope coefficient $(\hat{\alpha}_2)$ from the DOLS estimation together with the standard error s.e.$(\hat{\alpha}_2)$ computed using Newey and West's (1987) estimator of the long-run variance of the regression error. The estimates of interest semi-elasticity vary in a very small range from 1.69 to 1.75 throughout the sample lengths, which are very slightly higher than the estimates from the static regressions. In addition, the standard errors s.e.$(\hat{\alpha}_2)$ at each value of q are very small confirming the point estimates $(\hat{\alpha}_2)$ significantly different from zero. Overall, the results conclude that the stability of the Cagan (1956) money demand function linking ln(m) and r prevails and M1RS may be taken as the intermediate target of monetary policy for the post-1980 period while MZM may not be the right measure.

MONEY DEMAND EQUATION: JAPAN

In this section, this paper examines the functional form of money demand and its stability in the case of Japan. The above procedure is applied and M1 is used as the measure of money balances. A dummy variable *D06*, which is 1 after 2005:4, is included in all the regressions in which the samples extend into 2006. The purpose of variable is to allow for a possible break in the deterministic trends in money-income ratio and nominal interest rate. For those samples, critical values for Z_t with two explanatory variables are also provided under "Case 2" in Hamilton's (1994, 766) Table B.9.

Table 7 presents the Phillips and Ouliaris (1990) test for cointegration of money demand in Japan. The results reported are the intercept $(\hat{\alpha}_1)$ and slope coefficients $(\hat{\alpha}_2)$ from an OLS regression of equation (1) in the left panel and (2) in the right panel and the slope coefficient $(\hat{\rho})$ from a regression of the error term from equation (1) or (2) on its own lagged value.

The results point to the stability of log-log specification of money demand for the post 1980 period. The tests summarized in the left panel of Table 7 rejects the null hypothesis of no cointegration between ln(m) and ln(r) in almost all the sample lengths at least at the 10 percent significance level while none of the tests summarized in the right panel of Table 7 rejects the null hypothesis of no cointegration between ln(m) and r throughout the recursive samples. The functional form of money demand is consistent with that used in previous studies on Japan. In their recursive samples starting 1955-1974 and ending in 1990, Hoffman et al. (1995) reports an estimate of interest elasticity for Japan falling from 0.99 to 0.52 throughout the sample lengths. Our results report even smaller estimates, but rising from 0.09 in the 1980-2000 period to 0.12 for the sample ending in 2010.

In a robustness test to guarantee that the rejection of the semi-log form is not driven by the restriction of unitary income elasticity, a linear relationship linking real money balances ln(M/P) to real income ln(Y/P) and nominal interest rate r is estimated. Table 8 (see Web Appendix) reports the OLS estimates of the intercept $\hat{\beta}_1$, the slope coefficients $\hat{\beta}_2$ and $\hat{\beta}_3$ that measure the income elasticity and interest semi-elasticity of money demand, and the slope coefficient $(\hat{\rho})$ obtained from the regression of the error term on its own lagged value. At both lag truncation q = 4 and 8, the Phillips-Ouliaris statistic Z_t fails to reject the null hypothesis of no cointegration across the sample lengths. Taken together, the results rule out completely the semi-log specification for Japan by favoring the log-log specification of money demand for M1.

To confirm the result of stability of log-log functional form, the Stock and Watson (1993) dynamic OLS is applied. Table 9 (see Web Appendix) reports intercept $(\hat{\alpha}_1)$ and the slope coefficient $(\hat{\alpha}_2)$ from the DOLS estimation together with the standard error s.e.$(\hat{\alpha}_2)$ computed using Newey and West's (1987) estimator of the long-run variance of the regression error. The estimates of interest elasticity vary in a very small range from 0.10 to 0.14 throughout the sample lengths, which closely resemble the estimates from the static regressions. In addition, the very small standard errors s.e.$(\hat{\alpha}_2)$ at each value of q significantly reject the null hypothesis of zero point estimates $(\hat{\alpha}_2)$. The results point to the log-log form of Meltzer (1963) money demand and the stability of relationship linking ln(m) and ln(r) in the case of Japan.

MONEY DEMAND EQUATION: AUSTRALIA

It have been well known that after the financial deregulation in the 1980s, M3, which used to be the intermediate target of the Reserve Bank of Australia in implementing the monetary policy, has served no more a good measure of the demand for money (see Orden and Fisher, 1993). Several attempts have been made by researchers and the bank staffs alike to find the right measure of money demand. de Brouwer, Ng and Subbaraman (1993) employ Engle and Granger (1987) and Johansen (1988) techniques on the demand for M1, M3 and broad money, and find little evidence of the linear combination between money, income and interest rates.

In this section, this paper looks at the money demand in Australia. This study applies Phillips and Ouliaris (1990) cointegration technique as the above cases on the demand for M1. Like before, the current study examines the functional form of money demand and its stability. Table 10 presents the Phillips and Ouliaris (1990) test for cointegration of money demand for M1 in Australia. The results reported are the intercept $(\hat{\alpha}_1)$ and slope coefficients $(\hat{\alpha}_2)$ from an OLS regression of equation (1) in the left panel and (2) in the right panel and the slope coefficient $(\hat{\rho})$ from a regression of the error term from equation (1) or (2) on its own lagged value.

The results show that the money demand relationship linking ln(m) and r seem to be more stable than that linking ln(m) and ln(r). In the left panel, the Phillips-Ouliaris statistics Z_t can only rejects the null hypothesis of no cointegration between ln(m) and ln(r) in latter two of the sample lengths. On the other hand, the evidence of cointegrating relations between ln(m) and r (in the right panel) is significantly present in the latter half of the sample lengths starting from those extended into 2005 though it seems to break in the sample 1980-2007. The point estimates of the interest semi-elasticity are quite stable over the cointegrating sample lengths at about 5.74.

In a robustness test, a linear relationship linking real money balances ln(M/P) to real income ln(Y/P) and nominal interest rate ln(r) is estimated. Table 11 (see Web Appendix) reports the OLS estimates of the intercept $\hat{\beta}_1$, the slope coefficients $\hat{\beta}_2$ and $\hat{\beta}_3$ that measure the income elasticity and interest elasticity of money demand, and the slope coefficient $(\hat{\rho})$ obtained from the regression of the error term on its own lagged value. At both lag truncation q = 4 and 8, the Phillips-Ouliaris statistic Z_t cannot reject the null hypothesis of no cointegration across the sample lengths. In conclusion, the results point to M1 as the good measure of money demand in Australia. The money demand for M1 in the semi-log specification with unitary income elasticity restriction is quite stable, especially in the samples extended into 2005 through to 2010.

Table 12 (see Web Appendix) takes upon the success of our static results by applying the Stock and Watson (1993) dynamic OLS to the semi-log form of Cagan (1956) linking ln(*m*) and *r*. The table reports intercept $(\hat{\alpha}_1)$ and the slope coefficient $(\hat{\alpha}_2)$ from the DOLS estimation together with the standard error s.e.$(\hat{\alpha}_2)$ computed using Newey and West's (1987) estimator of the long-run variance of the regression error. The estimates of interest semi-elasticity vary from 5.49 to 6.45, which are quite similar to the estimates from the static regressions. In addition, the very small standard errors s.e.$(\hat{\alpha}_2)$ at each value of q significantly reject the null hypothesis of zero point estimates $(\hat{\alpha}_2)$. This paper argues that M1 is the good measure of money aggregate in the 2000s and the semi-log form with interest semi-elasticity of 5.74 is quite stable.

WELFARE COST OF INFLATION

Bailey (1956) defines the welfare cost as the difference between the loss of consumer surplus (integrating under the money demand curve as the interest rate rises from zero to $r > 0$) and the seigniorage revenue (*rm*). That is, the welfare cost function $w(r)$ is expressed as:

$$w(r) = \int_0^r m(x)dx - rm(r) \tag{3}$$

Lucas (2000) derives the measure of welfare cost of inflation under the two respective money demand functions as:

$$w(r) = A\left(\frac{\eta}{1-\eta}\right)r^{1-\eta} \tag{4}$$

when the demand money function takes the log-log form of equation (1) and

$$w(r) = \frac{B}{\xi}\left[1 - (1 + \xi r)e^{-\xi r}\right] \tag{5}$$

when the money demand function takes the semi-log form of equation (2). As discussed in Lucas (2000), the value $w(r)$ is the fraction of income an economy would require to give up so that the people are indifferent between living in a steady state with a positive interest rate r and otherwise living in an identical steady state with a zero interest rate.

Figure 2 plots the welfare cost functions of the three countries using the static results from the Phillips-Ouliaris (1990) cointegration relationship of the 1980-2010 sample and its corresponding functional form. Lucas (2000) assumes that the steady-state real interest rate is about 3 percent in the U.S. economy. Hence, the nominal interest rate of 3 percent (r = 0.03) prevails under the policy of zero inflation or price stability and r = 0.06 prevails under a policy of 3 percent annual inflation. For the sake of comparison, the paper assumes a similar rate for Japan and Australia. The curves imply a welfare cost of pursuing zero inflation policy against the Friedman (1996) rule for the U.S. at 0.012 percent, for Japan at 0.115 percent and for Australia at 0.213 percent. At the 6 percent nominal interest rate or 3 percent inflation the welfare cost as opposed to the Friedman (1969) rule of zero interest rate is 0.05 percent for the U.S., 0.20 percent for Japan and 0.70 percent for Australia.

Figure 3 plots the curves w(r) – w(0.03), the welfare cost relative to the cost of pursuing a zero inflation, for the three countries. The welfare gain from reducing the annual inflation from 10 percent to zero is equivalent to an increase in real income of 0.20 percent for the U.S., 0.30 percent for Japan and 2.55 percent for Australia.

CONCLUSIONS

The current study applies Phillips-Ouliaris (1990) cointegration technique to the recursive samples of the U.S., Japan and Australia over the period 1980-2010. The results for the U.S. show that the demand for M1RS is stable under the semi-log specification and the demand for MZM is not stable under both log-log and semi-log specifications. There is no evidence of cointegrating relationship in both measure of money stock when the assumption of unitary income elasticity is relaxed. The evidence here confirms the aggregate measure of money stock used by Ireland (2009) and rejects the claim by Carlson and Keen (1996) and Teles and Zhou (2005) that MZM can be used as the intermediate target for money policy.

The cointegration results for Japan support the stability of the log-log form of the demand for M1. The estimate of the interest elasticity ranges from 0.09 to 0.12 in the static regressions and slightly higher in the DOLS regressions. The log-log specification seems to reflect the infinite satiation of the demand for money at near zero interest rate as the interest rate in Japan has remained below 0.1 percent for the last 15 years. For Australia, the semi-log form seems to be more stable, at least for the last half of the sample lengths. The estimate of semi-interest elasticity is about 5.7 in the static results and about 6 in the DOLS regressions which quite resembles the estimate Lucas (2000) obtains from the 1900-1994 U.S. data in the semi-log specification.

Using the estimates of the stable money demand functions, the welfare cost functions imply that there is trivial welfare gain for the U.S. to move from zero inflation to the Friedman (1969) zero nominal interest rate rule for deflation while the gain for Japan is substantial. Lucas (2000) notes that this is due to the difference in the functional form of money demand. However, if the money demand of Australia which fits the data of interest rate as low as 3 percent can be used to predict the behavior at interest rates in the zero to 3 percent range, the gain from the semi-log specification of Australia's demand for M1 is even greater than that from the log-log specification of Japan's. This seems to reinforce Ireland's (2009) conclusion that the functional form may not matter very much, but whether or not the behavior of money demand at (near) zero interest rate is limited by a finite satiation is critical. This brings the analysis back to Figure 1 in which the question asks whether the U.S. would pursue the trail of Japan when the interest rate remains and will remain near zero for a longer period of time.

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APPENDICES

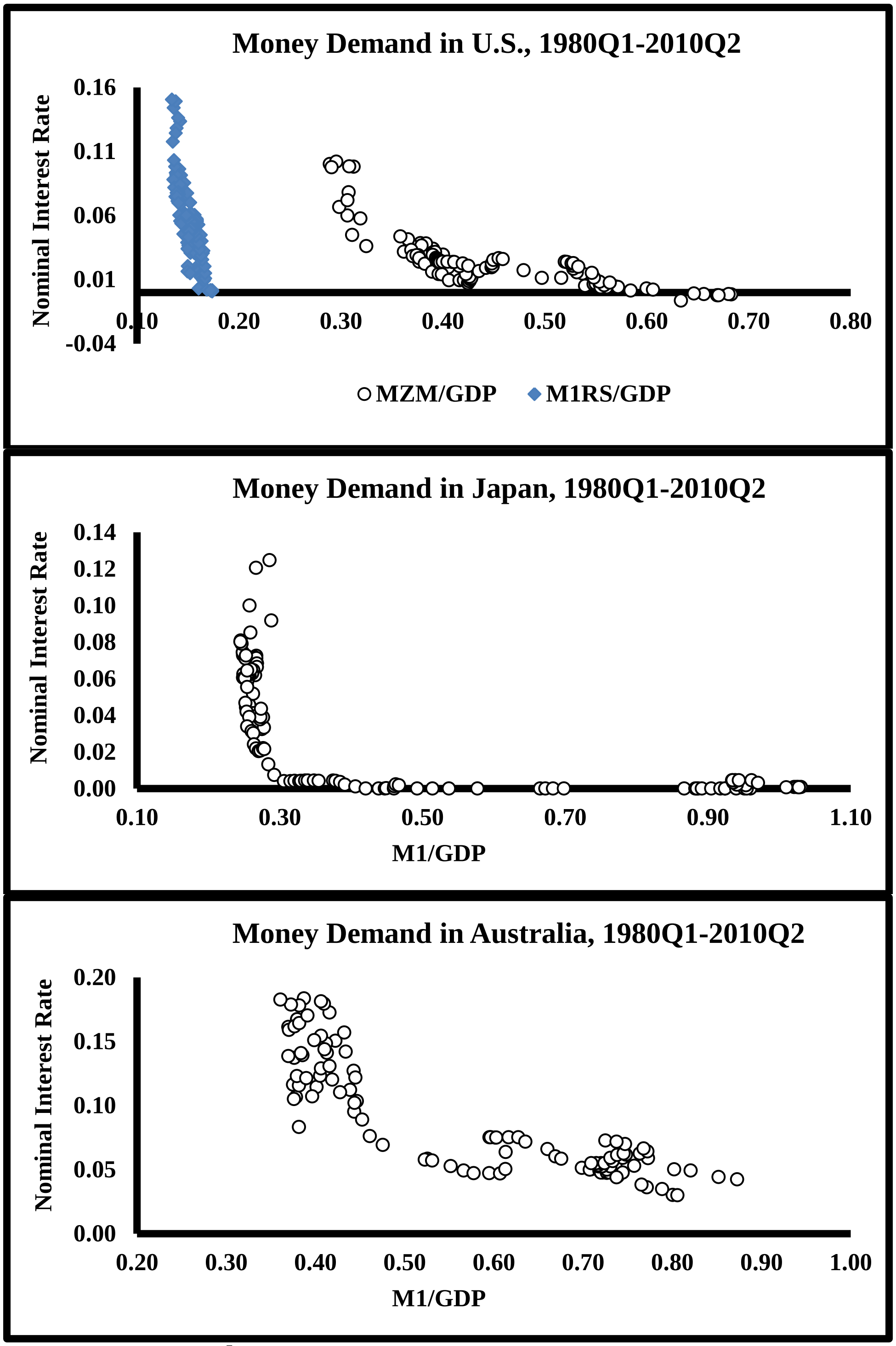

Figure 1: Money Demand, 1980-2010

Table 1: Phillips-Perron Unit Root Test Results: The United States

Variable	M1RS (1980:1-2010:2)				MZM (1980:1-2008:3)				
	$\hat{\mu}$	$\hat{\rho}$	q	Z_t	$\hat{\mu}$	$\hat{\rho}$	q	Z_t	
$\ln(m)$	-0.022	0.987	0	-0.636	0.002	0.996	0	-0.320	
			1	-1.053			1	-0.528	
			2	-1.306			2	-0.626	
			3	-1.489			3	-0.685	
			4	-1.596			4	-0.719	
			5	-1.658			5	-0.747	
			6	-1.698			6	-0.758	
			7	-1.708			7	-0.760	
			8	-1.692			8	-0.746	
$\ln(r)$	0.006	1.013	0	0.642	-0.213	0.954	0	-1.387	
			1	0.327			1	-1.499	
			2	0.119			2	-1.646	
			3	-0.097			3	-1.768	
			4	-0.318			4	-1.834	
			5	-0.416			5	-1.881	
			6	-0.470			6	-1.927	
			7	-0.506			7	-1.934	
			8	-0.517			8	-1.919	
r	0.001	0.953	0	-2.029	0.002	0.890	0	-3.493	**
			1	-2.118			1	-3.504	**
			2	-2.124			2	-3.496	**
			3	-2.143			3	-3.495	**
			4	-2.133			4	-3.493	**
			5	-2.129			5	-3.495	**
			6	-2.132			6	-3.495	**
			7	-2.102			7	-3.506	**
			8	-2.069			8	-3.529	***

Notes: Each panel reports $\hat{\mu}$ and $\hat{\rho}$, the intercept and slope coefficient from an ordinary least squares regression of the variable on a constant and its own lag, together with Z_t, the Phillips-Perron statistic corrected for autocorrelation in the regression error, computed using the Newey-West estimate of the error variance for various values of the lag truncation parameter q. The critical values for Z_t are reported by Hamilton (1994, Table B.6, 763): -2.58 (10 percent*), -2.89 (5 percent**), and -3.51 (1 percent***). m is money stock measured as the ratio of M1RS (left panel) and MZM (right panel) to GDP, respectively. r is the opportunity cost of holding money.

Table 2: Phillips-Perron Unit Root Test Results: Japan and Australia

Variable	Japan (1980:1-2010:2)				Australia (1980:1-2010:2)			
	$\hat{\mu}$	$\hat{\rho}$	q	Z_t	$\hat{\mu}$	$\hat{\rho}$	q	Z_t
ln(m)	0.019	1.009	0	1.903	0.002	0.997	0	-0.352
			1	1.620			1	-0.419
			2	1.462			2	-0.453
			3	1.358			3	-0.480
			4	1.223			4	-0.485
			5	1.113			5	-0.497
			6	1.029			6	-0.508
			7	0.961			7	-0.520
			8	0.904			8	-0.537
ln(r)	-0.159	0.978	0	-1.224	-0.057	0.980	0	-0.980
			1	-1.345			1	-1.222
			2	-1.430			2	-1.305
			3	-1.449			3	-1.348
			4	-1.463			4	-1.388
			5	-1.438			5	-1.400
			6	-1.414			6	-1.367
			7	-1.392			7	-1.344
			8	-1.387			8	-1.316
r	0.000	0.973	0	-1.776	0.002	0.970	0	-1.254
			1	-1.796			1	-1.417
			2	-1.804			2	-1.461
			3	-1.807			3	-1.504
			4	-1.807			4	-1.577
			5	-1.809			5	-1.617
			6	-1.808			6	-1.576
			7	-1.805			7	-1.551
			8	-1.804			8	-1.513

Notes: Each panel reports $\hat{\mu}$ and $\hat{\rho}$, the intercept and slope coefficient from an ordinary least squares regression of the variable on a constant and its own lag, together with Z_t, the Phillips-Perron statistic corrected for autocorrelation in the regression error, computed using the Newey-West estimate of the error variance for various values of the lag truncation parameter q. The critical values for Z_t are reported by Hamilton (1994, Table B.6, 763): -2.58 (10 percent*), -2.89 (5 percent**), and -3.51 (1 percent***). m is money stock measured as the ratio of M1 to GDP. r is the opportunity cost of holding money.

Table 3: Phillips-Ouliaris Cointegration Test Results: The United States

M1RS											
$\ln(m) = \alpha_1 - \alpha_2 \ln(r)$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\rho}$	q	Z_t	$\ln(m) = \alpha_1 - \alpha_2 r$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\rho}$	Q	Z_t
1980-2000	-2.242	0.121	0.891	4	-2.630		-1.797	1.606	0.864	4	-2.838
				8	-2.810					8	-2.982
1980-2001	-2.215	0.111	0.915	4	-2.222		-1.801	1.566	0.866	4	-2.868
				8	-2.472					8	-3.019
1980-2002	-2.168	0.093	0.934	4	-2.101		-1.802	1.554	0.867	4	-2.947
				8	-2.355					8	-3.096 *
1980-2003	-2.132	0.080	0.944	4	-2.061		-1.800	1.580	0.865	4	-3.031
				8	-2.295					8	-3.184 *
1980-2004	-2.128	0.078	0.936	4	-2.330		-1.796	1.631	0.865	4	-3.041
				8	-2.522					8	-3.206 *
1980-2005	-2.131	0.080	0.940	4	-2.318		-1.793	1.660	0.864	4	-3.134 *
				8	-2.538					8	-3.296 *
1980-2006	-2.130	0.080	0.937	4	-2.432		-1.792	1.666	0.862	4	-3.248 *
				8	-2.636					8	-3.405 **
1980-2007	-2.129	0.080	0.939	4	-2.483		-1.794	1.648	0.872	4	-3.122 *
				8	-2.706					8	-3.305 *
1980-2008	-2.092	0.066	0.983	4	-1.632		-1.801	1.562	0.882	4	-3.156 *
				8	-2.040					8	-3.308 *
1980-2009	-2.050	0.052	0.966	4	-1.867		-1.796	1.635	0.876	4	-3.372 **
				8	-2.113					8	-3.400 **
1980-2010	-2.041	0.049	0.953	4	-2.210		-1.793	1.667	0.873	4	-3.450 **
				8	-2.410					8	-3.477 **

Notes: Each panel reports $\hat{\alpha}_1$ and $\hat{\alpha}_2$, the intercept and slope coefficient from the ordinary least squares regression of $\ln(m)$ on $\ln(r)$ or r, recursively; $\hat{\rho}$, the slope coefficient from an ordinary least squares regression of the corresponding regression error on its own lagged value; and Z_t, the Phillips-Ouliaris statistic for $\rho = 1$ (the null hypothesis of no cointegration), corrected for autocorrelation in the residual, computed using the Newey-West estimate of the error variance for the lag truncation parameter q = 4 and 8. The critical values for Z_t are reported by Hamilton (1994, Table B.9, 766): -3.07 (10 percent*), -3.37 (5 percent**), and -3.96 (1 percent***).

Table 4: Phillips-Ouliaris Cointegration Test Results: The United States

MZM

$\ln(m) = \alpha_1 - \alpha_2 \ln(r)$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\rho}$	q	Z_t
1980-2000	-1.532	0.159	0.916	4	-1.332
				8	-1.488
1980-2001	-1.569	0.170	0.878	4	-2.274
				8	-2.398
1980-2002	-1.617	0.184	0.868	4	-2.371
				8	-2.522
1980-2003	-1.636	0.190	0.859	4	-2.589
				8	-2.739
1980-2004	-1.653	0.195	0.880	4	-2.169
				8	-2.235
1980-2005	-1.653	0.197	0.931	4	-1.498
				8	-1.595
1980-2006	-1.634	0.195	0.954	4	-1.160
				8	-1.273
1980-2007	-1.635	0.197	0.933	4	-1.956
				8	-2.048
1980-2008	-1.595	0.186	0.889	4	-2.542
				8	-2.655

Notes: $\hat{\alpha}_1$ and $\hat{\alpha}_2$, the intercept and slope coefficient are obtained from the ordinary least squares regression of $\ln(m)$ on $\ln(r)$, recursively; $\hat{\rho}$, the slope coefficient from an ordinary least squares regression of the corresponding regression error on its own lagged value; and Z_t, the Phillips-Ouliaris statistic for $\rho = 1$ (the null hypothesis of no cointegration), corrected for autocorrelation in the residual, computed using the Newey-West estimate of the error variance for the lag truncation parameter q = 4 and 8. The critical values for Z_t are reported by Hamilton (1994, Table B.9, 766): -3.07 (10 percent*), -3.37 (5 percent**), and -3.96 (1 percent***).

Table 7: Phillips-Ouliaris Cointegration Test Results: Japan

$\ln(m) = \alpha_1 - \alpha_2 \ln(r)$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\rho}$	q	Z_t		$\ln(m) = \alpha_1 - \alpha_2 r$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\rho}$	q	Z_t
1980-2000	-1.609	0.093	0.857	4	-2.579			-1.070	4.162	0.986	4	-0.809
				8	-1.876						8	-1.057
1980-2001	-1.610	0.094	0.744	4	-3.873	**		-1.024	4.874	0.999	4	-0.426
				8	-3.256	*					8	-0.677
1980-2002	-1.625	0.098	0.715	4	-4.051	***		-0.960	5.863	1.018	4	0.044
				8	-3.485	**					8	-0.213
1980-2003	-1.662	0.109	0.786	4	-3.436	**		-0.885	7.034	1.025	4	0.399
				8	-2.641						8	0.056
1980-2004	-1.691	0.117	0.817	4	-3.317	*		-0.816	8.110	1.008	4	-0.129
				8	-2.776						8	-0.391
1980-2005	-1.712	0.123	0.836	4	-3.208	*		-0.755	9.059	1.000	4	-0.407
				8	-2.714						8	-0.620
1980-2006	-1.701	0.121	0.680	4	-4.302	**		-0.755	9.060	0.948	4	-1.936
				8	-4.096	**					8	-2.042
1980-2007	-1.691	0.120	0.654	4	-4.766	***		-0.756	9.059	0.948	4	-1.978
				8	-4.437	***					8	-2.080
1980-2008	-1.688	0.119	0.643	4	-4.986	***		-0.756	9.058	0.947	4	-2.023
				8	-4.695	***					8	-2.127
1980-2009	-1.688	0.119	0.639	4	-5.128	***		-0.755	9.062	0.946	4	-2.075
				8	-4.825	***					8	-2.182
1980-2010	-1.688	0.119	0.640	4	-5.164	***		-0.755	9.063	0.946	4	-2.098
				8	-4.860	***					8	-2.206

Notes: Each panel reports $\hat{\alpha}_1$ and $\hat{\alpha}_2$, the intercept and slope coefficient from the ordinary least squares regression of $\ln(m)$ on $\ln(r)$ or r, recursively; $\hat{\rho}$, the slope coefficient from an ordinary least squares regression of the corresponding regression error on its own lagged value; and Z_t, the Phillips-Ouliaris statistic for $\rho = 1$ (the null hypothesis of no cointegration), corrected for autocorrelation in the residual, computed using the Newey-West estimate of the error variance for the lag truncation parameter q = 4 and 8. The critical values for Z_t are reported by Hamilton (1994, Table B.9, 766): -3.07 (10 percent*), -3.37 (5 percent**), and -3.96 (1 percent***). A dummy variable, *D06*, which is 1 after 2005Q4, is included in all the regressions in which the samples extend into 2006. For those, the critical values for Z_t are reported by Hamilton (1994, Table B.9, 766): -3.45 (10 percent*), -3.77 (5 percent**), and -4.31 (1 percent***).

Table 10: Phillips-Ouliaris Cointegration Test Results: Australia

$\ln(m) = \alpha_1 - \alpha_2 \ln(r)$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\rho}$	q	Z_t		$\ln(m) = \alpha_1 - \alpha_2 r$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\rho}$	q	Z_t	
1980-2000	-1.856	0.482	0.916	4	-2.110			-0.221	4.765	0.893	4	-2.408	
				8	-2.143						8	-2.351	
1980-2001	-1.906	0.507	0.904	4	-2.421			-0.180	5.051	0.894	4	-2.477	
				8	-2.451						8	-2.430	
1980-2002	-1.925	0.516	0.886	4	-2.762			-0.159	5.206	0.873	4	-2.880	
				8	-2.785						8	-2.832	
1980-2003	-1.935	0.520	0.885	4	-2.824			-0.146	5.300	0.871	4	-2.953	
				8	-2.837						8	-2.896	
1980-2004	-1.947	0.527	0.884	4	-2.892			-0.134	5.381	0.870	4	-3.036	
				8	-2.900						8	-2.973	
1980-2005	-1.957	0.532	0.884	4	-2.956			-0.124	5.447	0.869	4	-3.119	*
				8	-2.967						8	-3.054	*
1980-2006	-1.970	0.539	0.892	4	-2.864			-0.112	5.526	0.872	4	-3.111	*
				8	-2.878						8	-3.043	
1980-2007	-1.983	0.547	0.909	4	-2.646			-0.097	5.620	0.881	4	-3.037	
				8	-2.676						8	-2.972	
1980-2008	-1.989	0.552	0.898	4	-2.938			-0.085	5.688	0.877	4	-3.195	*
				8	-2.957						8	-3.134	*
1980-2009	-1.951	0.536	0.896	4	-3.103	*		-0.080	5.722	0.873	4	-3.331	*
				8	-3.069						8	-3.262	*
1980-2010	-1.946	0.534	0.895	4	-3.156	*		-0.079	5.734	0.872	4	-3.370	*
				8	-3.098	*					8	-3.300	*

Notes: Each panel reports $\hat{\alpha}_1$ and $\hat{\alpha}_2$, the intercept and slope coefficient from the ordinary least squares regression of ln(m) on ln(r) or r, recursively; $\hat{\rho}$, the slope coefficient from an ordinary least squares regression of the corresponding regression error on its own lagged value; and Z_t, the Phillips-Ouliaris statistic for $\rho = 1$ (the null hypothesis of no cointegration), corrected for autocorrelation in the residual, computed using the Newey-West estimate of the error variance for the lag truncation parameter q = 4 and 8. The critical values for Z_t are reported by Hamilton (1994, Table B.9, 766): -3.07 (10 percent*), -3.37 (5 percent**), and -3.96 (1 percent***).

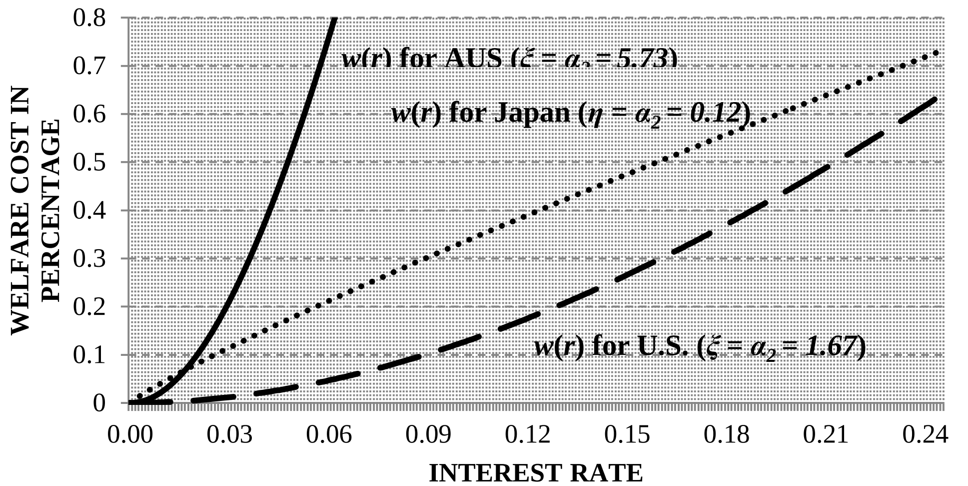

Figure 2: Welfare cost functions

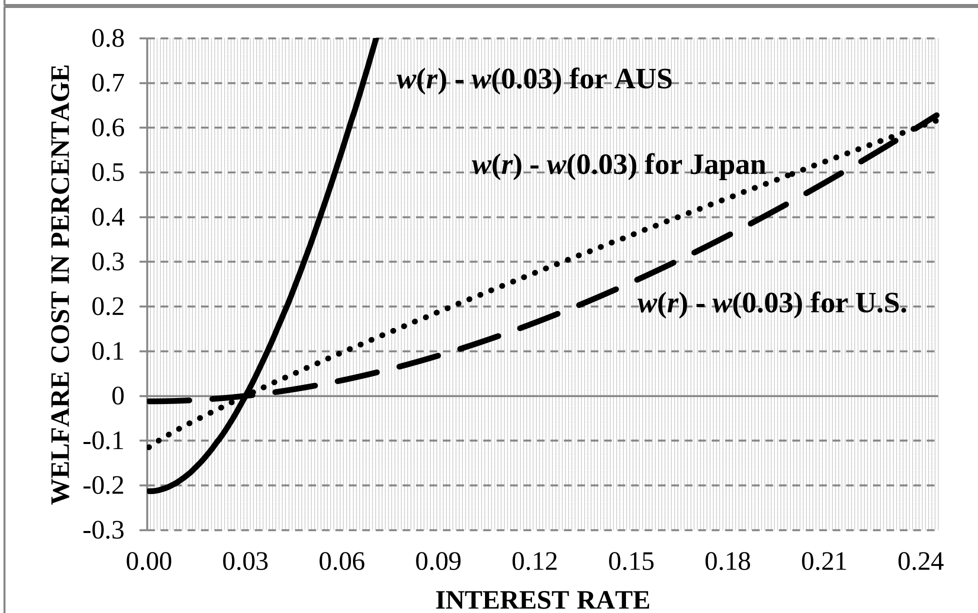

Figure 3: Welfare cost relative to 3% interest rate

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[i] Bailey (1956) defines the welfare cost as the difference between the loss of consumer surplus (integrating under the money demand curve as the interest rate rises from zero to $r > 0$) and the seigniorage revenue (rm). This will be discussed in details in the later section after we obtain the empirical results.

[ii] MZM (Money zero maturity) = M2 – Small-denomination time deposits + Institutional money market mutual funds.

[iii] Web Appendix can be found in the full version of the paper on SSRN at http://ssrn.com/abstract=2032761

[iv] We apply Phillips and Ouliaris (1990) test for cointegration to the semi-log specification linking ln(M/P), ln(Y/P) and r with lag truncation q = 4 and 8. The Z_t statistics cannot reject the null hypothesis of no cointegration in all sample lengths. The results are not reported, but available upon request. Hence, it is not estimated in the dynamic OLS, either.